Multiplication

Objective 1

Understand the Meaning and Notation of Multiplication

There are different ways to indicate multiplication. Here are a few.

$3 \times 4 \quad 3 \cdot 4 \quad 3(4) \quad (3)(4)$

$$\begin{array}{r} 3 \\ \times\ 4 \\ \hline \end{array}$$

But what does $3 \cdot 4$ or "3 times 4" actually mean?

$$3 \cdot 4 = 3 + 3 + 3 + 3 = 12$$

So what does $24 \cdot 10$ or "24 times 10" actually mean?

$24 \cdot 10 = 24 + 24 + 24 + 24 + 24 + 24 + 24 + 24 + 24 + 24$

$24 \cdot 10 = 240$

Notice there are ten 24's being added together!

Now, how can we easily calculate $24 \cdot 11$?

Since we know that $24 \cdot 10 = 240$, and that $24 \cdot 10$ represents ten 24's being added together, we just need to add one more 24 to 240!

$24 \cdot 10 = 240$

$24 \cdot 11 = 240 + 24 = 264$

Notice that when using the vertical format to calculate $24 \cdot 11$, you end up having to find the sum of 24 and 240!

$$24 \cdot 11 = 240 + 24 = 264$$

$$\begin{array}{r} 24 \\ \times\ 11 \\ \hline 24 \\ +240 \\ \hline 264 \end{array}$$

Note: The vertical format uses the "Distributive Property" and the "Expanded Form" of a number.

$$24 \cdot 11 = 24(\underbrace{10+1}_{\text{The Expanded Form of 11}}) = 24(10+1) = 240 + 24 = 264$$

Example 1: Use the distributive property and the expanded form of a number to perform each multiplication problem.

a) $18 \cdot 11 = 18(10+1) =$

b) $9 \cdot 11 = 9(10+1) =$

c) $7 \cdot 12 = 7(10+2) =$

d) $5 \cdot 13 = 5(10+3) =$

e) $13 \cdot 12 = 13(10+2) =$

f) $8 \cdot 7 = 8(5+2) =$

Now, how can we easily calculate $24 \cdot 9$? Since we know that $24 \cdot 10 = 240$, and that $24 \cdot 10$ represents ten 24's being added together, we just need to subtract 24 from 240!

$$24 \cdot 11 = 240 + 24 = 264$$

$$24 \cdot 10 = 240$$

$$24 \cdot 9 = 240 - 24 = 216$$

Objective 2

Understand the Multiplication Table

There are certain multiplication problems that we can sometimes easily recall, like $8 \cdot 5 = 40$. Knowing this, we can now use a pattern to calculate other multiplication problems with the number 8 as well as with other numbers!

$8 \cdot 5 = 40$	$9 \cdot 5 = 45$	$12 \cdot 5 = 60$
$8 \cdot 6 = 48$	$9 \cdot 6 = 54$	$12 \cdot 6 = 72$
$8 \cdot 7 = 56$	$9 \cdot 7 = 63$	$12 \cdot 7 = 84$
$8 \cdot 8 = 64$	$9 \cdot 8 = 72$	$12 \cdot 8 = 96$
$8 \cdot 9 = 72$	$9 \cdot 9 = 81$	$12 \cdot 9 = 108$
$8 \cdot 10 = 80$	$9 \cdot 10 = 90$	$12 \cdot 10 = 120$
$8 \cdot 11 = 88$	$9 \cdot 11 = 99$	$12 \cdot 11 = 132$

Example 2: Construct a 12 by 12 multiplication table and write down any patterns that you notice.

Objective 3

Perform Multiplication using the Vertical Format

Recall that the vertical format is based on using the distributive property and the expanded form of a number.

$$36 \cdot 23 = 36(20+3) = 36(20+3) = 720 + 108$$

The Expanded Form of 23

Example 3: Calculate $36 \cdot 23$ using the vertical format.

First multiply 36 by 3.

$$\begin{array}{r} {\scriptstyle 1}\,36 \\ \times\quad 3 \\ \hline 108 \end{array}$$

Next, multiply 36 by 20.

$$\begin{array}{r} {\scriptstyle 1}\,36 \\ \times\ 20 \\ \hline 720 \end{array}$$

Finally add the results

$$\begin{array}{r} 36 \\ \times\ 23 \\ \hline 108 \\ +\,720 \\ \hline 828 \end{array}$$

Answer the following homework questions.

In Exercises 1 – 9, perform each multiplication problem.

1) $9 \cdot 4$
2) $9 \cdot 5$
3) $9 \cdot 6$
4) $7 \cdot 5$
5) $7 \cdot 6$
6) $7 \cdot 7$
7) $6 \cdot 4$
8) $6 \cdot 5$
9) $6 \cdot 6$

In Exercises 10 – 15, first rewrite each multiplication problem as an addition problem, then find the sum.

Example: $3 \cdot 4 = 3 + 3 + 3 + 3 = 12$

10) $5 \cdot 4$
11) $6 \cdot 4$
12) $9 \cdot 3$
13) $8 \cdot 6$
14) $12 \cdot 4$
15) $150 \cdot 4$

In Exercises 16 – 21, perform each multiplication problem using the vertical format.

16) $17 \cdot 9$
17) $12 \cdot 11$
18) $37 \cdot 15$
19) $45 \cdot 12$
20) $60 \cdot 20$
21) $55 \cdot 24$

In Exercises 22 – 27, write in the correct number to make the equation true.

22) $9 \cdot \_\_ = 54$
23) $\_\_ \cdot 9 = 36$
24) $7 \cdot \_\_ = 42$
25) $\_\_ \cdot 8 = 24$
26) $12 \cdot \_\_ = 108$
27) $\_\_ \cdot 6 = 72$

Objective 4: Write a mathematical expression using words.

Definition

The **product** of two numbers **a** and **b** is written **a** · **b**. The word **product** indicates multiplication.

Example 4: Using the word **product**, write "9 · 7" as a word statement, and find the value of the product.

We first begin our sentence by defining the mathematical operation first and then define the numbers. Notice how the word "and" is used.

The word statement is written as:

"The product of nine and seven."

The value of the product is 63.

Example 5: Using the word **product**, write "16 · 7" as a word statement, and find the value of the product.

Answer the following homework questions.

28) The word "product" is used to represent _____.

29) Write "the product of 12 and 7" using math symbols.

30) Write "the product of p and q" using math symbols.

31) Using the word product, write "19 • 14" as a word statement and find the value of the product.

In Exercises 32 – 35, find each product.

32) 28×30

33) 52×27

34) 320×15

35) 600×400